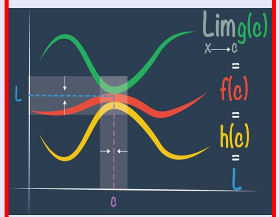
Calculus I

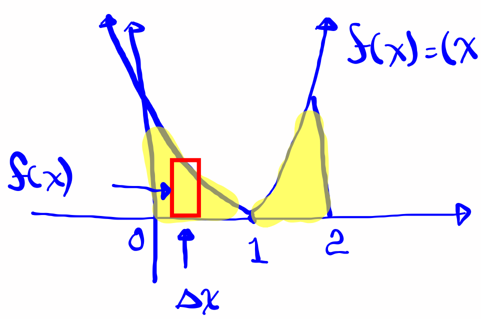
Lecture 54



Feb 19-8:47 AM

find the area of region bounded by $f(x)=(x-1)^2$, and x-axis from $x=0$ to $x=2$.

Parabola upward



$$A=\int_0^2 (x-1)^2\,dx$$

$u=x-1$ $\quad$ $x=0 \rightarrow u=-1$

$du=dx$ $\quad$ $x=2 \rightarrow u=1$

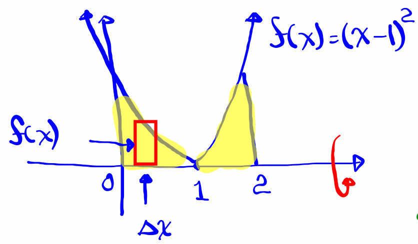
$$A=\int_0^2 (x-1)^2\,dx=\int_{-1}^{1} u^2\,du = 2\int_0^1 u^2\,du = 2\cdot\frac{u^3}{3}\Big|_0^1=\boxed{\frac{2}{3}}$$

even function

May 22-8:45 AM

Find the volume when region bounded by $f(x)=(x-1)^2$, and x-axis from $x=0$ to $x=2$ is rotated about x-axis.

Parabola upward



1) Ref. Rect. ⊥ A.O.R.

2) Region is totally attached to A.O.R

Disk Method

$$V=\int_a^b \pi\left[f(x)\right]^2 dx = \int_0^2 \pi\left[(x-1)^2\right]^2 dx = \pi\int_0^2 (x-1)^4 dx$$

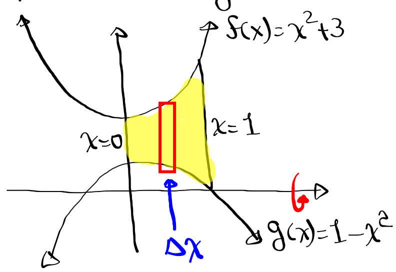
$u=x-1$

$$=\pi\int_{-1}^{1} u^4\,du = 2\pi\int_0^1 u^4\,du$$

$$=2\pi\cdot\frac{u^5}{5}\Big|_0^1 = \boxed{\frac{2\pi}{5}}$$

May 22-8:45 AM

Find the volume of the region shaded below and rotated by x-axis.



1) Ref. Rect. ⊥ A.O.R.

2) Region is not totally attached to A.O.R.

Washer Method

$$V=\int_a^b \pi\left[Top^2 - Bottom^2\right]dx$$

Top $= x^2+3$, Bottom $= 1-x^2$

$$V=\int_0^1 \pi\left[(x^2+3)^2-(1-x^2)^2\right]dx$$

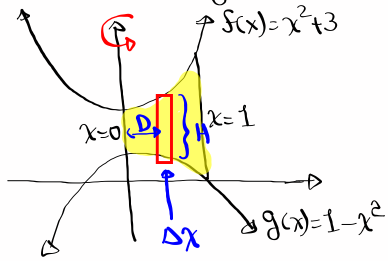
$$=\pi\int_0^1\left[\cancel{x^4}+6x^2+9-1+2x^2-\cancel{x^4}\right]dx$$

$$=\pi\int_0^1\left[8x^2+8\right]dx = 8\pi\left[\frac{x^3}{3}+x\right]\Big|_0^1 = 8\pi\left(\frac{1}{3}+1\right)$$

$$=8\pi\cdot\frac{4}{3}=\boxed{\frac{32\pi}{3}}$$

May 22-8:57 AM

Find the volume of the region shaded below and rotated by y-axis.



Ref. Rect. is now parallel to A.O.R.

Shell Method

$$V=\int_a^b 2\pi \cdot D \cdot H \, dx$$

D is the distance from A.O.R. $D=x$

H is the height of Ref. Rect. $H=\text{Top}-\text{Bottom}$

$$=x^2+3-(1-x^2)$$
$$=2x^2+2$$

$$V=\int_0^1 2\pi \, x\,(2x^2+2)\,dx$$

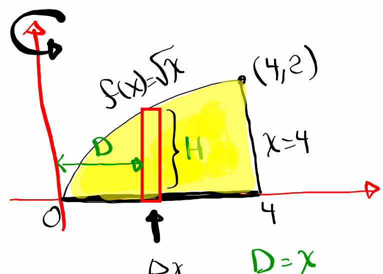
$$=4\pi\int_0^1 (x^3+x)\,dx = 4\pi\left[\frac{x^4}{4}+\frac{x^2}{2}\right]\Bigg|_0^1 = 4\pi\left[\frac{1}{4}+\frac{1}{2}\right]$$

$$=4\pi\cdot\frac{3}{4}=\boxed{3\pi}$$

May 22-8:57 AM

Rotate the region bounded by $f(x)=\sqrt{x}$, $g(x)=0$, and $x=4$ about the y-axis.

Find the volume.



Ref. Rect. is parallel to A.O.R.

$\Rightarrow$ Shell Method

$D=x$

$H=\sqrt{x}-0$

$$V=\int_a^b 2\pi\cdot D\cdot H \quad dx$$

$$=\int_0^4 2\pi\, x\sqrt{x}\, dx = 2\pi\int_0^4 x^{\frac{3}{2}}dx = 2\pi\cdot\frac{x^{5/2}}{5/2}\Bigg|_0^4$$

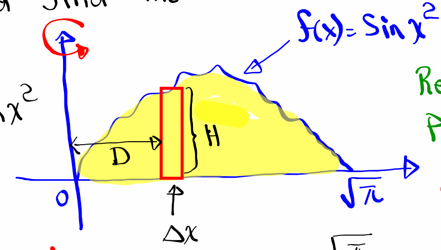
$$=2\pi\cdot\frac{2}{5}\, x^2\sqrt{x}\Bigg|_0^4 = \frac{4\pi}{5}\cdot 4^2\sqrt{4}$$

$$=\boxed{\frac{128\pi}{5}}$$

May 22-9:13 AM

Rotate the region shaded below by Y-axis, and find the volume.

$D = x$
$H = \sin x^2$



Ref. Rect. is Parallel to A.O.R.

Shell Method

$$V = \int_a^b 2\pi \, D \, H \, dx = \int_0^{\sqrt{\pi}} 2\pi \, x \, \sin x^2 \, dx$$

$u = x^2$
$du = 2x\,dx$

$x = 0 \rightarrow u = 0^2 = 0$
$x = \sqrt{\pi} \rightarrow u = (\sqrt{\pi})^2 = \pi$

$$= \int_0^{\pi} \pi \sin u \, du = \pi \cdot -\cos u \Big|_0^{\pi}$$

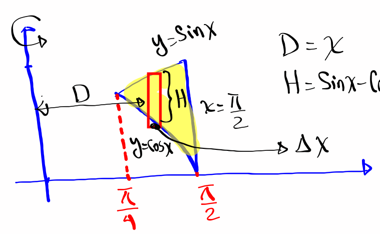
$$= -\pi \left[\cos\pi - \cos 0 \right] \quad (\cos\pi = -1,\ \cos 0 = 1)$$

$$= -\pi(-2) = \boxed{2\pi}$$

May 22-9:22 AM

Rotate the region enclosed by $x = \frac{\pi}{4}$, $x = \frac{\pi}{2}$, $y = \sin x$, and $y = \cos x$ about Y-axis.

Find the Volume.



$D = x$
$H = \sin x - \cos x$

Ref. Rect. is parallel to A.O.R.

Shell Method

$$V = \int_a^b 2\pi \, D \, H \, dx = 2\pi \int_{\pi/4}^{\pi/2} x(\sin x - \cos x)\,dx$$

$$= 2\pi \int_{\pi/4}^{\pi/2} \left[x \sin x - x \cos x \right] dx$$

Integration by Parts
Calc II
Table of integration

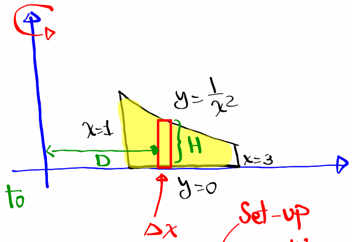
$$\int x \sin x \, dx = \sin x - x\cos x + C$$

$$\int x \cos x \, dx = \cos x + x \sin x + C$$

May 22-9:33 AM

Rotate the region bounded by $x=1$, $x=3$, $y=0$, and $y=\frac{1}{x^2}$ about Y-axis.

Find the volume.



$D = x$

$H = \frac{1}{x^2}$

Ref. Rect. is parallel to A.O.R.

Shell Method

Set-up only

$$V = \int_a^b 2\pi D H\, dx = 2\pi \int_1^3 x \cdot \frac{1}{x^2}\, dx = \boxed{2\pi \int_1^3 \frac{1}{x}\, dx}$$

$\frac{1}{x} \to x^{-1}$

$$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \text{if } n \neq -1}$$

using table of integrations or wait for Calc. II

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int_1^3 \frac{1}{x}\, dx = \ln x \Big|_1^3 = \ln 3 - \cancel{\ln 1}^{\,0} = \ln 3$$

Final Ans.

$$\boxed{2\pi \ln 3}$$

May 22-9:45 AM